AP Calculus AB
Chapter 5 Practice

Name Key

Find the equation of the function that passes through the given point.

1. $\frac{dy}{dx} = 3x^2 + 1$ through (1,4)

$\int dy = \int (3x^2+1)\,dx$

$y = x^3 + x + C$

$(4) = (1)^3 + (1) + C$

$2 = C$

$y = x^3 + x + 2$

2. $f'(x) = \frac{x}{2y}$ if $f(2) = 3$

$\frac{dy}{dx} = \frac{x}{2y}$

$\int 2y\,dy = \int x\,dx$

$y^2 = \frac{1}{2}x^2 + C$

$(3)^2 = \frac{1}{2}(2)^2 + C$

$7 = C$

$y^2 = \frac{1}{2}x^2 + 7$

$y = \pm\sqrt{\frac{1}{2}x^2+7}$ ← extraneous

$y = \sqrt{\frac{1}{2}x^2+7}$

3. An object has an acceleration function of $a(t) = t^2 - 3t - 1$. The object has a velocity of 13 feet per second at six seconds. It has a position of –2.25 feet at three seconds.

a) What are the position and velocity functions?

$v(t) = \int t^2 - 3t - 1\,dt$

$v(t) = \frac{1}{3}t^3 - \frac{3}{2}t^2 - t + C$

$(13) = \frac{1}{3}(6)^3 - \frac{3}{2}(6)^2 - (6) + C$

$13 = 72 - 54 - 6 + C$

$13 = 12 + C$

$1 = C$

$v(t) = \frac{1}{3}t^3 - \frac{3}{2}t^2 - t + 1$

$s(t) = \int \frac{1}{3}t^3 - \frac{3}{2}t^2 - t + 1\,dt$

$s(t) = \frac{1}{12}t^4 - \frac{1}{2}t^3 - \frac{1}{2}t^2 + t + C$

$(-2.25) = \frac{1}{12}(3)^4 - \frac{1}{2}(3)^3 - \frac{1}{2}(3)^2 + (3) + C$

$-2.25 = \frac{81}{12} - \frac{27}{2} - \frac{9}{2} + 3 + C$

$-2.25 = -8.25 + C$

$6 = C$

$s(t) = \frac{1}{12}t^4 - \frac{1}{2}t^3 - \frac{1}{2}t^2 + t + 6$

b) What are the initial velocity and position of the object?

$v(0) = 1$ ft/sec

$s(0) = 6$ ft.

c) When the object is at rest what is the object's position?

$v(t) = 0 \quad s(t) = ?$

$0 = \frac{1}{3}t^3 - \frac{3}{2}t^2 - t + 1$

$t = .572$ sec

$t = 4.981$

$s(.572) = 6.324$ ft

$s(4.981) = -11.918$ ft.

d) What are the acceleration, velocity, and position of the object at 5.6 seconds?

$a(5.6) = 13.56$ ft/se

$v(5.6) = 6.899$ ft/sec

$s(5.6) = -9.934$ ft

e) What are the total distance traveled and the displacement of the object in the first six seconds?

displacement:

$\int_0^6 \frac{1}{3}t^3 - \frac{3}{2}t^2 - t + 1 \, dt$

-12 ft.

total distance:

$\int_0^6 |v(t)| \, dt$

24.484 ft.